NPS55-86-005

NAVAL POSTGRADUATE SCHOOL

Monterey, California

PROJECT REPORT

SUBSUB - A SUBMARINE ENGAGEMENT MODEL

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MARCH 1986

Prepared for:
Naval War College
Newport, EI 02841

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NPS55-86-005	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b. OFFICE SYMBOL (If applicable) Code 55	7a. NAME OF MONITORING ORGANIZATION Naval War College
6c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000	7b. ADDRESS (City, State, and ZIP Code) Newport, RI 02841
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N0012486WR35588
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
SUBSUB - A SUBMARINE ENGAGEMENT MODEL

12. PERSONAL AUTHOR(S)
Eagle, James N.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
	FROM 1 Oct 85 TO 31 Dec 85	1986, March	25

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	submarine, model, engagement,

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

An examination is made of the submarine versus submarine engagement model SUBSUB. Engagements are modelled from search through detection, classification, localization, closure, attack and counterattack. The submarine missions considered are barrier patrol, area patrol and transit. The version of SUBSUB examined runs on an IBM-PC microcomputer. Other versions exist which operate on larger computers.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL James N. Eagle	22b. TELEPHONE (Include Area Code) (408)646-2654 22c. OFFICE SYMBOL Code 55Er

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

SUBSUB - A Submarine Engagement Model

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Introduction

This document reports the results of an examination of the submarine engagement model SUBSUB, written by Presearch, Inc. of Fairfax, VA, and modified for use at the Naval War College (NWC), Newport, RI . This model considers a submarine versus submarine interaction from search through detection, classification, localization, closure, attack, and counterattack. The submarine missions modelled in are barrier patrol, area patrol, and transit. The version of SUBSUB examined runs on an IBM-PC. Other versions exist which operate on larger computers.

Purpose of this Study

The primary purpose of the examination was to assess the modelling methods used in SUBSUB and to judge the program's usefulness for seminar war gaming at NWC. This evaluation was conducted using documentation and FORTRAN source code supplied by Presearch, Inc.

A secondary purpose of the examination was to describe the basic models involved and suggest possible improvements or extensions.

Limits of this Study

SUBSUB is an extensive model, or more accurately, an integrated collection of models. The examination considered in detail the available

documentation and FORTRAN source code for the search, detection, and closure modules. The environmental and platform data bases were not examined.

SUBSUB Overview

SUBSUB models one-on-one barrier, area, and transit search missions for submarine platforms. It is an analytical model, as opposed to a discrete time step simulation. The submarine can be assigned a variety of current or projected sonar systems and weapon capabilities. Each platform is modelled in either an aggressive or evasive posture. Countermeasures are allowed. Each encounter begins with detection and continues through classification, localization, closure, attack, and counterattack. Figure-of-merit inputs (target source levels and environmental parameters) are drawn from a disk-based data base. The version of SUBSUB examined contained only winter environments, but others could presumably be added as necessary.

The program begins with the user reviewing the last selections for environment, scenario, platforms, sensors, and weapons. Any or all of these selections can be changed. The possible scenarios for the opposing submarines (called Blue and Red) are: barrier search/transit, area search/transit, area search/area search, transit/transit, and direct support/penetrator.

When the user is satisfied with the initial conditions, the program calculations are begun. Outputs of the program include all input data, nominal detection and counterdetection ranges (apparently the maximum range at which mean signal excess is 0), and a sequence of calculated probabilities for Blue and Red detection, closure, attack,

counterattack, and kill. Random variables uniformly distributed between 0 and 1 are compared to each calculated probability. If the random variable is less than the probability, the event in question is said to occur. The model then progresses to the next event in the engagement event tree. For example, if it is determined that Blue achieves a secure detection, the next probability calculated is that of correct classification by Blue, followed by closure, localization, and attack.

Detailed Model Description

Probability Density for Detection Range. For the selected location and season, a propagation loss curve is retrieved from a disk-based database. Using the passive sonar equation a mean figure-of-merit ($\overline{FOM}$) is calculated. The actual figure-of-merit is assumed to be

$$FOM = \overline{FOM} + \xi,$$

where ξ is a normally distributed random variable with mean 0 and standard deviation specified by the program. Using the selected propagation loss curve, a detection range is associated with each value of FOM. In this manner a density function for detection range is determined. (Figure 1.)

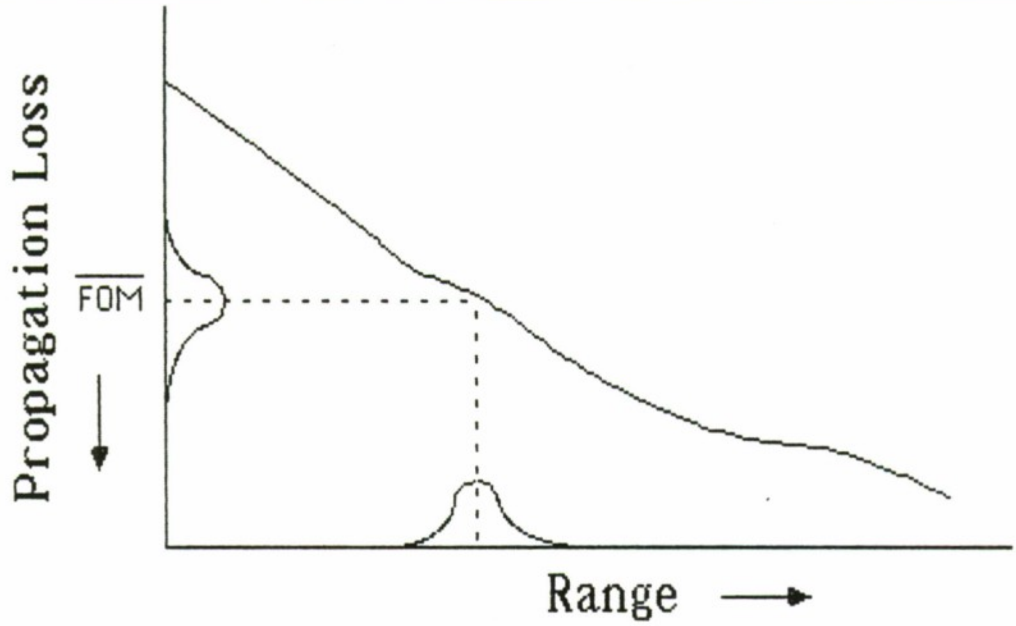

Figure 1. Probability Density Function for Detection Range.

The same calculation is done for the target, resulting in a density function for counterdetection range. Although it is not clear from the documentation or source code, the range associated with each FOM is most likely the maximum range such that propagation loss equals FOM.

An implicit assumption of this model is that detection and counterdetection ranges are fixed during the entire encounter at values determined by probability distributions. This is in contrast to discrete time step computer simulations using lambda-sigma or Gauss-Markov detection models, where time-varying fluctuations in the environment cause these ranges to change over time.

<u>Distribution of Minimum Range Achieved during Search.</u> If $\rho(t)$ is the minimum range achieved during the search from time 0 to time t, then $P_{CPA}(t,R)$ is defined as

$$\text{Prob}\{\rho(t) \leqslant R\}.$$

The functional form of P_{CPA} changes with the geometry of the encounter. In the barrier search problem, P_{CPA} is assumed only a function of R. That is, $P_{CPA}(R)$ is the probability that the minimum range achieved during the barrier penetration is R or less. The calculation of P_{CPA} for barrier and area search geometries will be discussed in detail later.

<u>Probability of Secure Detection ($D\overline{CD}$).</u> The probability of secure detection is computed by conditioning on the detection range of the searcher. Specifically, if $f_{R_D}(r)$ is the density function for detection range and R_{CD} is counterdetection range, then the probability of achieving a secure detection by time t is

$$\int_0^\infty \text{Prob}\{R_{CD} \leqslant r\}\, f_{R_D}(r)\, P_{CPA}(t,r)\, dr. \qquad (1)$$

In SUBSUB, $D\overline{CD}$ is evaluated by conditioning on the searcher FOM. Letting $r(\nu)$ be the detection range associated with FOM ν and $f_{FOM}(\nu)$ be the probability density function for FOM, then $D\overline{CD}$ becomes

$$\int_{-\infty}^\infty \text{Prob}\{R_{CD} \leqslant r(\nu)\}\, f_{FOM}(\nu)\, P_{CPA}(t,r(\nu))\, d\nu. \qquad (2)$$

This expression is integrated numerically. The probability of a secure counterdetection is computed similarly.

This method of calculating the probability of a secure detection has much to recommend it. As opposed to the procedures of reference [1], this method incorporates the relative motion between the searcher and

target. Thus the probability of achieving a secure detection in an area search mission will in general be different from that in a barrier mission. This occurs because the distribution of CPA ranges is different for the two cases.

The disadvantage of this approach, however, is the requirement to perform a numerical integration. This slows the execution of the program. But it should be noted that even the simpler, geometry-independent method of reference [1] requires numerical integration (or extensive table look up). It appears that this is the numerical price that must be paid for secure detection calculations when detection and counterdetection ranges are random variables.

If searcher has a significant acoustic advantage, then these calculations are probably not necessary. In this case, the probability of secure detection by time t approximately equals the probability of detection by time t. But when detection and counterdetection capabilities are nearly equal, then it becomes more important to carefully consider the target's potential to counterdetect.

<u>Barrier search P_{CPA}.</u> From the SUBSUB documentation, $P_{CPA}(R)$ for a searcher conducting a barrier patrol against a target with a constant course and speed is

$$(2R/(BW \sin\beta)) - (R^2/(BW\,T \tan^2\beta)) + (2A_1/(BW\,T \tan\beta)) \qquad (3)$$

where

$$A_1 = ((90-\beta)/360)\,\pi R^2 \quad \text{if } R < (BW-T)/2$$

or

$$A_1 = .5((BW-T)/2)^2 \tan\sigma + (90-\beta-\sigma)\pi R^2/360 \quad \text{if } (BW-T)/2 < R < (BW-T)/(2\sin\beta)$$

or

$$A_1 = .5((BW-T)/2)^2\,(1/\tan\beta) \quad \text{if } R > (BW-T)/(2\sin\beta).$$

The other definitions are:

T = width of searcher's track,

BW = barrier width (the target's penetration is uniformly distributed over this length),

V_0 = searcher speed,

V_T = target speed,

$\beta = \tan^{-1}(V_T/V_0)$,

$\sigma = \cos^{-1}((BW-T)/(2R))$.

This expression is a modification of the "standard" calculations in Koopman's reference [2]. The general computational method used here is to consider the barrier penetration in "target-stationary relative space". That is, the target is assumed to be stationary, and all relative speed for the encounter is provided by the searcher. The searcher's relative speed component across the barrier front is the searcher's actual speed, V_0. And the component of relative speed perpendicular to the barrier is the target speed, V_T. Then the probability that the target comes within range R of the searcher during a barrier penetration is the ratio of the area "covered" by the searcher in relative space during one

pass across the barrier to the total area that could be occupied by the target. In Figure 2., this covered area is shaded.

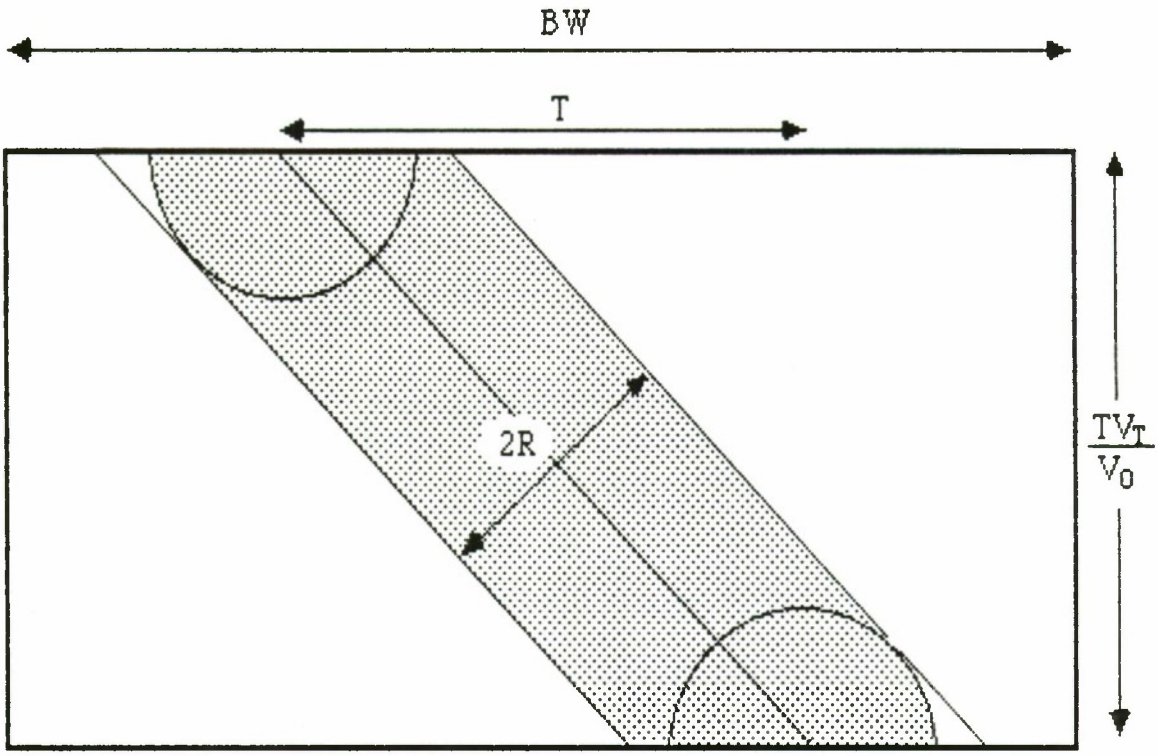

Figure 2. Calculation of P_{CPA} for Barrier Search.

There are several problems with (3). The first is that (3) approaches negative infinity as V_T becomes small or R becomes large. The physical explanation for this is that the method used to calculate the area of the shaded region of Figure 2. (i.e., dividing the region into parallelograms, triangles and sections of circles) fails for small V_T or large R. The mathematical reason for this behavior is that the second additive term in (3) dominates the other two terms in these limiting cases.

By way of example, Figure 3. is a plot of P_{CPA}, as calculated by (3), for target speeds, V_T, of 1 to 10 knots. Here R is 18 nautical miles (nm), V_0 is 10 knots, BW is 100 nm, and T is 60 nm. Since the actual P_{CPA} should be a decreasing function of V_T, the calculated values for V_T less than about 2.5 knots are suspect.

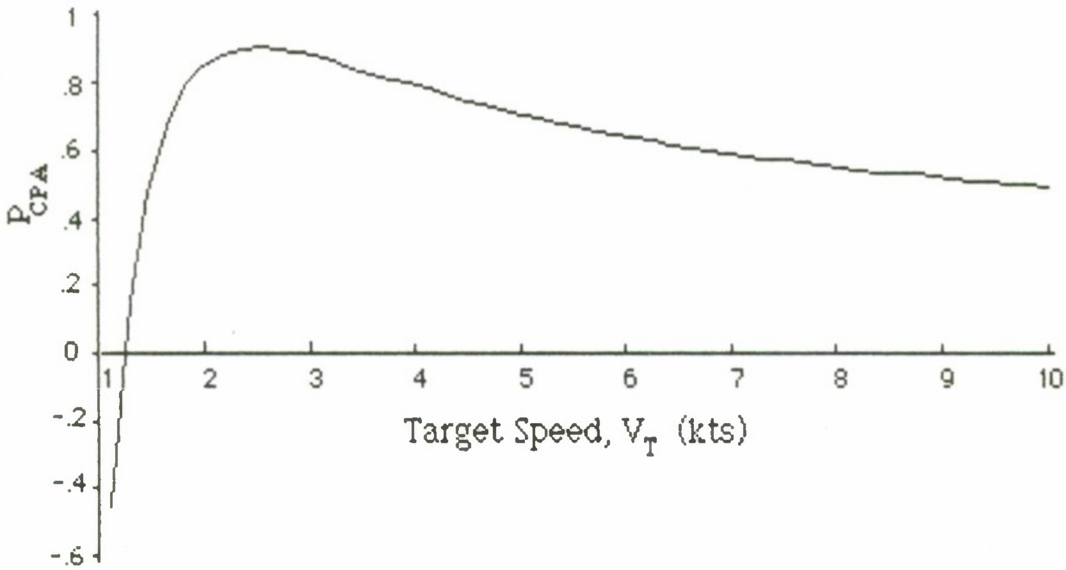

Figure 3. SUBSUB P_{CPA} vs. V_T.

Figure 4. is a plot of P_{CPA} vs. range, R. In this example, V_T is 5 knots; R varies from 20 to 60 nm; and V_0, BW, and T are as before. Actual P_{CPA} should be an increasing function of R (as long as $P_{CPA} < 1$, at least) but is seen here to decrease for R greater than about 36 nm.

Although not mentioned in the supplied documentation, the SUBSUB code (in subroutine ACPA) modifies (3) so as to reduce the problem created by large values of R. Specifically, when $R > (BW-T)/(2 \sin \beta)$, then P_{CPA} is calculated with R set to $\min\{R, (T \tan^2\beta)/\sin\beta\}$. This change prevents P_{CPA} from approaching negative infinity, but can introduce a discontinuity into the function and does not prevent P_{CPA} from decreasing with increasing R.

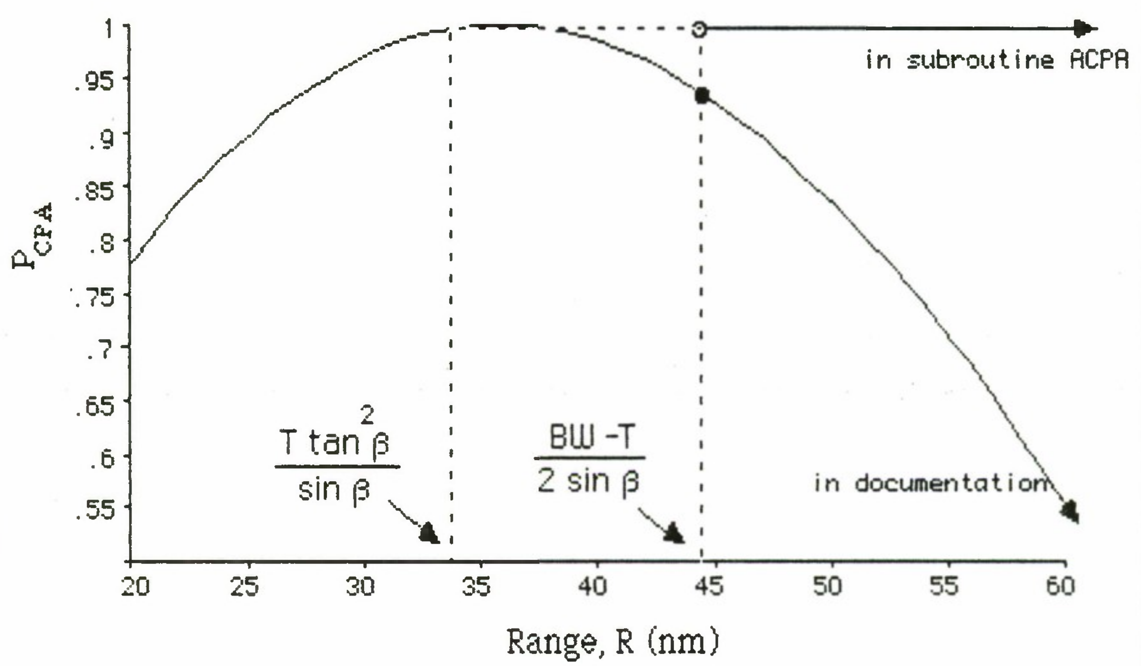

Figure 4. SUBSUB P_{CPA} vs. R as in documentation and source code.

Another problem with the approach used to derive (3) is that it is not applicable for BW < T. In fact, the BW terms in the denominators of the additive terms of (3) cause the calculated P_{CPA} to become infinite as BW approaches 0. The SUBSUB program does not warn the user that BW < T is not allowed.

A third (and relatively minor) problem with (3) is that if the methodology use to compute A1 in the first two cases is followed for the third case (i.e., when $R > (BW-T)/(2\sin\beta)$), then A1 should be

$$.5\{ ((BW-T)/2)^2 (1/\tan\beta) - (\tan\beta)(R - (BW-T)/(2\sin\beta))^2 \}.$$

However, as discussed above, using this methodology at all is not recommended.

Alternative analytical barrier models are available which avoid these problems and are more general in their application. A good example is the JOTS II (Joint Operational Tactical System) barrier model described in [4]. This model generalizes slightly the computational method in Koopman [2] to estimate $P_{CPA}(x)$, the probability of coming within range R of a transitor when the barrier penetration point is x. This probability is given as

$$P_{CPR}(x) = \min\{f(x), 1\} \tag{4}$$

where

$$f(x) = \begin{cases} 2RK/T & \text{for } |x| \leqslant T/2 - RK \\ (RK/T) + (T/2 - |x|)/T & \text{for } T/2 - RK \leqslant |x| \leqslant T/2 + R/K \\ (1/T)\sqrt{(R^2 - (|x| - T/2)^2)(K^2 - 1)} & \text{for } T/2 + R/K \leqslant |x| \leqslant T/2 + R \\ 0 & \text{for } |x| \geqslant T/2 + R \end{cases}$$

$$K = \sqrt{1 + (V_0/V_T)^2},$$

and x is 0 at the center of the searcher track T.

If the barrier penetration point is given by a probability distribution, $g_X(x)$, then

$$P_{CPR} = \int_{-(T/2)-R}^{(T/2)+R} P_{CPR}(x)\, g_X(x)\, dx. \tag{4.1}$$

Currently, the SUBSUB model implicitly assumes that the barrier penetration point is a uniformly distributed random variable with mean 0. Using (4.1) allows any distribution of penetration points. For example, an attractive strategy for the target would be to attempt to maximize $|x|$ (that is, penetrate as far as possible from the center of the barrier), since f(x) is nonincreasing in $|x|$.

When the assumption of a uniform distribution of penetration points with mean 0 is acceptable, then it is possible to closely approximate (4.1) with a closed form expression. There are three cases:

$$P_{CPA} = \begin{cases} \min\{2RK/T, 1\} & \text{for } BW \leq T - 2RK \\ 1 - \{(1/BW\,T)\,[\max\{.5(T+BW) - RK, 0\}]^2\} & \text{for } T-2RK \leq BW \leq T+2R \\ ((T+2R)/BW)\ (1 - [\max\{T - R(K-1), 0\}]^2/(T^2+2RT)) & \text{for } BW \geq T+2R \end{cases}$$

where K is as before.

Barrier Search Probability of Closure (P_{CL}). The SUBSUB documentation describes two different methods for estimating P_{CL}, which is the probability of successfully closing a target detected during barrier search. The first method appears to compare the time required for the searcher to approach to within weapons range using a bearing rider track (i.e., the relative bearing of the target is always 000°) to the time required for the target to escape out the bottom of the search area. The angle AOB_{MAX} is calculated, which is the maximum angle-on-the-bow at detection allowing closure to weapon range before the target escapes. Then based on a geometrical construction which is equivalent to assuming a cosine distribution of target AOB at detection, the probability that the actual AOB will be less than AOB_{MAX} is determined. This is taken to be P_{CL}. Although appearing in the documentation, this method was not implemented in the version of SUBSUB received for review.

In the SUBSUB code (subroutine BCLOSE), a more direct method is used to determine AOB_{MAX}. But then AOB_{MAX} is converted to P_{CL} in a nonstandard manner.

The program assumes that the searcher requires a specified length of time (variable TIM) to conduct target motion analysis (TMA).

Immediately after detection, the searcher heads for where the target will be at the end of the TMA period. (The documentation differs from the code by stating that the searcher closes down the initial target bearing.) At the end of the TMA period, the searcher takes a corrected intercept course for target closure. If the searcher can close to within weapon range before the target exits the barrier area, then the initial AOB at detection is determined to be small enough to allow closure. The program iterates through various initial AOB's (first in 10° then in 1° increments) until the maximum AOB allowing closure is found.

This much is straightforward. However, the program then converts AOB_{MAX} into a probability of closure in a fashion that requires further justification.

One "standard" assumption (which is implicitly used by the first procedure described in the documentation) is that initial AOB will have a cosine distribution. Specifically, reference [3] gives the probability density function of AOB at detection as

$$f(\theta) = \begin{cases} .5\cos(\theta - \gamma), & -\pi/2 + \gamma \leqslant \theta \leqslant \pi/2 + \gamma \\ 0, & \text{otherwise} \end{cases}$$

where $\gamma = \tan^{-1}(V_0/V_T)$ is the mean AOB at initial detection. Given a density function for AOB and a constant AOB_{MAX}, P_{CL} can be determined by integrating the density function from $-AOB_{MAX}$ to AOB_{MAX} (which is particularly simple for the distribution $f(\theta)$ given above).

In SUBSUB, P_{CL} is computed using $f(\theta)$ when the searcher detection range is sufficiently small (specifically, less than $T/(20\sin(AOB_{MAX}))$. And for larger detection ranges, P_{CL} increases linearly to a maximum value,

beyond which P_{CL} is constant. The reasoning behind this is (apparently) that the assumption of a cosine density for initial AOB is a poor one for large enough detection ranges. This is reasonable, but just how large is "large enough" and how P_{CL} will increase requires more justification than currently appears in the documentation.

To conclude, the subroutine BCLOSE in SUBSUB appears to give believable values for P_{CL}, but the underlying model is somewhat ad hoc. Further analytical justification or validation with simulation results seems appropriate here.

<u>P_{CPA} for Area Search</u>. SUBSUB calculates P_{CPA} as follows:

$$P_{CPA} = 1 - (1 - (\pi R^2/A)) \exp(-2RV_r t/(A-\pi R^2)) \tag{6}$$

where

A = search area size,

$V_r = (V_O^2 + V_T^2)^{1/2}$ = an "effective" relative speed,

R = detection range.

Problems with (6) include:

1. Target and searcher are assumed to be searching in the same area of ocean. This seems a very unreasonable assumption.

2. V_r, the relative speed between the searcher and target, is calculated assuming that the target and searcher velocity vectors are perpendicular. The more standard assumption is that the angle between the velocity vectors is uniformly distributed between 0 and 2π. Then by the law of cosines,

$$V_r = (1/2\pi) \int_0^{2\pi} (V_0^2 + V_T^2 - 2V_0V_T\cos\theta)^{1/2}\, d\theta$$

$$= (1/\pi) \int_0^{\pi} (V_0^2 + V_T^2 - 2V_0V_T\cos\theta)^{1/2}\, d\theta \qquad (7)$$

Evaluation of (7) using numerical integration might not be advisable on a microcomputer. However, equation (7) is can be approximated by the following closed form expression:

$$V_r \approx .38 \max(V_0, V_T) + .62\, (V_0^2 + V_T^2)^{1/2}\,.$$

3. Implicit in (6) is the assumption that the area searched the instant the search begins never need be searched again. In effect, the search area reduces in size from A to $(A-\pi R^2)$ at time 0^+. This is consistent with a target stationary in relative space but is inconsistent with the assumption of a moving target which can migrate into areas previously searched.

An alternative expression addressing the above concerns is

$$P_{CPA}(t) = (1 - (\pi R^2 A_{ST})/(A_S A_T))\, \exp(-2RV_r A_{ST} t/(A_S A_T)), \qquad (8)$$

where A_S is the size of the searcher's patrol area, A_T is the size of the target's patrol area, and A_{ST} is the size of area common to both the searcher and the target. Also, V_r is given by (7).

It is noted that neither (6) nor (8) model convergence zones. As a minimum modification for convergence zone propagation, the πR^2 term in (8) should be replaced with A_{CZ}, which is the area of ocean giving convergence zone and direct path detections. For example, if detections are possible at ranges of 0-4, 30-33 and 61-63 nm from the searcher, then

$$A_{CZ} = \pi 4^2 + \pi(33^2-30^2) + \pi(63^2-61^2) \text{ nm}^2$$
$$= 1423 \text{ nm}^2$$

Probability density function for tracking range. Equation (1.25) of the SUBSUB documentation gives the density function for tracking range when the searcher has two independent sensors as

$$\phi_0(R) = \phi_{D1}(R) + \phi_{D2}(R) - \phi_{D1}(R) \int_{-\infty}^{R} \phi_{D2}(R')\, dR', \tag{9}$$

where $\phi_{D1}(R)$ is the density function for tracking range for sensor 1 and $\phi_{D2}(R)$ is the density for the tracking range of sensor 2. (Actually, the documentation gives the lower limit of integration as positive infinity, but this is assumed to be a typographical error.) Equation (9) is not a proper density function since it integrates to a value greater than 1 (with either sign of the lower limit of integration). The proper form for (9) is

$$\phi_0(R) = \phi_{D1}(R) + \phi_{D2}(R) - \phi_{D1}(R) \int_{-\infty}^{R} \phi_{D2}(R')\, dR' - \phi_{D2}(R) \int_{-\infty}^{R} \phi_{D1}(R')\, dR' . \tag{10}$$

Probability of Achieving Track given no Counterdetection ($\overline{TCD}$). In the SUBSUB documentation, $\overline{TCD}$ is calculated by conditioning on the range required to achieve "track" on the target. In (1.27) of the documentation, $\overline{TCD}$ is expressed as

$$\int_0^{R_{max}} (1-P_{CD}(R))\, \phi_T(R)\, dR$$
$$= \text{Prob}\{R_{CD} < R_{TRACK}\}, \tag{11}$$

where $\phi_T(R)$ is the density function for tracking range, R_{CD} is counterdetection range and R_{TRACK} is tracking range. Given the event tree structure of the overall program, it appears as if the correct probability should be conditioned on the searcher achieving a secure detection. That is, $\overline{TCD}$ should be

$$\text{Prob}\{R_{CD} < R_{TRACK} \mid R_{CD} < R_D\}. \quad (12)$$

As long as the target has some secure detection capability (i.e., $\text{Prob}\{R_{CD} < R_D\} < 1$), equation (12) gives a value strictly greater than that given by equation (11).

A suggested simplification of the SUBSUB model would be to assume that $\overline{TCD}$ is some user specified probability, say .8 or .9, depending on the searcher's capability to conduct passive target motion analysis. This seems a reasonable assumption and would reduce the number of numerical integrations performed.

Attack Probabilities. Equation (1.28) of the SUBSUB documentation is an expression for the probability of a secure attack ($\overline{FCD}$). This probability is calculated by conditioning on the attacker's launch range. The equation is correct except that the integration should be performed from minimum launch range to maximum launch range. This same apparent error occurs in Figure 1-6 of the documentation.

Convergence Zone (CZ) Environments. CZ environments are modelled in a nonstandard fashion in SUBSUB. It does appear, however, that the final effect of CZ propagation on the SUBSUB calculations might be small. In CZ environments, P_{CPA} is modified as follows:

$$P_{CPA}(R) = \begin{cases} P_{CPA}(R) & \text{if } R<R_I \\ (P_{CPA}(R) + P_{CPA}(R_{CZ}))/2 & \text{if } R \geq R_I \end{cases} \tag{13}$$

Here R_I is the maximum direct path detection range, and R_{CZ} is the range to the first CZ. The justification given in the documentation is that initial CZ detections will make subsequent redetection easier, so P_{CPA} should be larger between direct path and CZ ranges. The result of (13) is to insert a discontinuity in P_{CPA} at range R_I. See Figure 5.

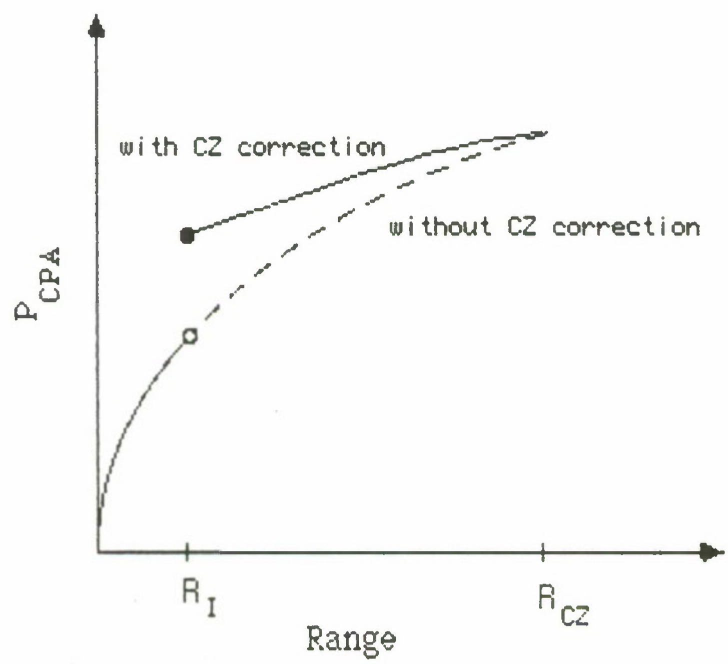

Figure 5. P_{CPA} in a Convergence Zone Environment.

This modification of P_{CPA} appears odd considering that in non-CZ environments P_{CPA} is a function of the searcher and target tracks under the assumption that detection has not yet occurred. That is, P_{CPA} reflects only search tactics and not approach tactics. Also the discontinuity at range R_I is difficult to justify intuitively. Furthermore, (13) apparently applies in environments with only one CZ. The

generalization to the multiple-CZ case is not given in the SUBSUB documentation.

It does appear that the effect of this modification of P_{CPA} might have a relatively small impact on SUBSUB calculations, at least in the calculation of the probability of a secure detection ($\overline{DCD}$). This probability is given by

$$\int_0^\infty \text{Prob}\{R_{CD} \leqslant r\}\, f_{R_D}(r)\, P_{CPA}(t,r)\, dr, \qquad (14)$$

where $f_{R_D}(r)$ is the density function for detection range. In a CZ environment, $f_{R_D}(r)$ will be very near 0 for ranges between R_I and R_{CZ}, since most initial detections will occur either in the CZ or at the maximum direct path range. But it is for those ranges between R_I and R_{CZ} that P_{CPA} is increased. So it <u>appears</u> that the CZ modification to P_{CPA} may, in fact, change the evaluation of (14) only slightly. This conclusion might change, however, depending on how the multiple-CZ modification is accomplished.

Analytical Models and Discrete Time Step Simulations

SUBSUB is an integrated collection of analytical models. For such models to determine whether or not a particular event occurs, an expression for the probability of that event is evaluated. Then a random number, uniformly distributed between 0 and 1, is drawn. If the random number is less than or equal to the calculated probability, then the event is said to occur. This is opposed to discrete time step simulations (e.g., SIM II, IBGTT and ENWGS), where the coordinates of the

individual platforms are moved during each time step and the status of each platform (e.g., course, speed, depth, radiated and self noise levels) is updated. An advantage of analytical models is that, in addition to determining whether or not the event in question occurs <u>this time</u>, the calculated probability tells the user how likely an occurrence of the event will be <u>next time</u> and all subsequent times. This could be important if the user needs to know whether a particular outcome is a low probability event or something to be expected regularly.

An important limitation of analytical models is the possible lack of verified and validated submodels to calculate the required probabilities. For example, in SUBSUB one of the ASW missions modelled is systematic area search for a randomly patrolling target. There exists no simple, generally accepted analytical model for this search scenario. What is used in SUBSUB is a modified version of Koopman's random search expression, even though the search tracks modelled may be anything but random. It this a reasonable thing to do? The answer depends on the particulars of the situation being modelled. If the relative motion is dominated by the target, then random search is probably a good assumption. If, on the other hand, the target speed is slow relative to the searcher, then a systematic search model would be more appropriate.

Or consider barrier search. The model used in SUBSUB to compute the probability of detecting a transitor assumes (1) a constant speed search, (2) the target track is orthogonal to the searcher track, and (3) the barrier penetration point is uniformly distributed around a mean position at the center of the searcher track. If it is desired to model any other case then SUBSUB may not be appropriate.

Or if environmental conditions are changing with position or time (such as in the marginal ice zone), then SUBSUB would probably not be the best choice.

Discrete time step simulations, on the other hand, are more robust and can generally consider more types of scenarios. If, for example, it is desired to know if a transitor successfully penetrates a barrier when the searcher conducts a sprint and drift search, the physical situation is established (initial positions, courses, speed, ranges, and depths) and one replication of the simulation is run. Either detection occurs or it does not. An advantage of this method is that the modelling can often be simple, geometric, and straightforward (compared to analytical models which frequently become rather esoteric). The disadvantage of this method is that it does not give a probability of event occurrence unless many replications are performed. But for seminar wargames, probabilities are often of secondary importance. What is of primary concern is typically whether or not a particular event happens.

To conclude this section, it is suggested that there are probably some missions and scenarios modelled in SUBSUB which could be handled in a more straightforward manner with a discrete time step simulation.

Conclusions

SUBSUB is a large program. It is a collection of analytical models which address the phases of one-on-one submarine engagements from search through detection, classification, closure, attack and counterattack. And it appears to run reliably on an IBM-PC microcomputer.

The basic structure of the program is a conditional event tree, which is hard to fault. However, some of the component models do have limitations or errors which should be addressed in the documentation and subsequent revisions of the program. This report mentions some of these problems, but there may be others which escaped notice.

In spite of the program's shortcomings, it is judged to be better than the microcomputer programs previously used at NWC to evaluate submarine engagements. It is recommended that the current version of SUBSUB be used by NWC, but that the known errors be corrected and the technical documentation formalized as soon as feasible.

One separate but related issue briefly raised was whether analytical models or discrete time step simulations are more appropriate for seminar wargames. There is probably a place for both. If analytical models are used, however, it is very important to know the modelling assumptions coded into the expression for probability of event occurrence. These assumptions might be more restrictive than one would like. Compared to analytical models, simulations tend to be more robust and transparent to the user. However, they can take longer to run and do not give the probability of event occurrence.

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